Assessing Student Understanding:
A Framework for Testing and Teaching

What is assessment? To many noneducators, the term is synonymous with “test” or “quiz.” However, teachers use the word to describe any method of gathering information about student learning. Whether it be formative assessment (intended to guide instructional decisions) or summative assessment (a reflection on the entirety of student learning from prior instruction), teachers are constantly working to identify what their students know and how deeply they understand what they are learning.

The process of assessing student learning requires teachers to look beyond merely correct and incorrect responses that students have given to different questions and assessment items. If a student makes an error, teachers must discern the severity of the error and what caused the incorrect response. Was it a small computational error? Did the student misread the question or not understand what was being asked in class? Or is there a deeply held misconception that will make learning all future related content very difficult without some type of remediation? These are the types of questions that teachers frequently ask themselves during classroom instruction and while reviewing student work.
Via vignettes, look inside first- and fourth-grade classrooms where teachers demonstrate how to use a research-based structure during instruction to choose tasks that elicit different levels of comprehension.
A single problem can have a range of difficulty because of variances in student proficiency.

The four levels of reasoning that comprise the assessment framework can be used at multiple junctures.

**IDMT**

Initiative for Developing Mathematical Thinking (IDMT) is a professional development and research organization dedicated to supporting teachers of mathematics around the United States through professional development and technical assistance (Brendefur et al. 2015). Part of our work with school districts often focuses on assessment. IDMT staff frequently review teacher-made assessments and published curricular materials. IDMT staff also work side by side with teachers in the classroom to enhance questioning strategies for the purpose of formatively assessing students’ learning.

**IDMT assessment framework**

The IDMT assessment framework comprises three levels of assessment and draws on more substantive work in the field of task and assessment from such authors as Hiebert (1986), Webb (2002), de Lange (1999), and Stein and Smith (1998). Initially, we examined Stein and Smith’s 1998 framework on tasks, which focuses on lower-level demands, such as memorization and procedures without connections, and higher-level demands, such as procedures with connections and doing mathematics. Extending their framework on tasks, we combined two national and international frameworks on assessment: (1) Webb’s (2007) depth of knowledge framework, used by both the Partnership for Assessment of Readiness for College and Careers (PARCC) as well as the Smarter Balanced Assessment Consortium (SBAC), focuses on recalling information, engaging in mental processing, strategic thinking, and extended thinking; and (2) de Lange’s (2002) work on assessment, which culminated with the PISA framework, integrates the mathematical topic, level of difficulty, and goal levels. The first goal is reproduction work, followed by connections and concepts, and finally, reasoning. We incorporated these frameworks to build an assessment framework to include levels of reasoning that teachers must attend to:

- Level 1: Skill, rote, and procedural
- Level 2: Problem solving
- Level 2: Conceptual
- Level 3: Justification

These levels of reasoning have been used in districts to build end-of-unit assessments and as a daily framework for asking questions or gathering formative assessment. In a three-year Institute of Education Sciences (IES) grant, we demonstrated that instructional practices coupled with this assessment framework improved student achievement on standardized measures.
For teachers, the framework is helpful specifically because it allows them to assess student learning on the same content at these different levels. In this way, teachers can determine how thoroughly students understand the content. The likelihood of uncovering students’ misconceptions also increases as teachers examine student responses by reflecting on the different levels of questions. The framework then informs teachers directly regarding how they might modify future lessons or assessment materials on the basis of which question levels students demonstrate their needs on particular content.

The framework has three levels (see fig. 1). Note the four different categories; Level 2 includes two different types of assessment.

**Level 1: Skill**

Items from the Level 1: Skill category typically require students to produce a response by recalling information or following a set of rote procedures. For example, using a standard addition algorithm to solve 146 + 58 is considered a Level 1: Skill item. Another example would be to ask students to provide the definition of a parallelogram or of a prime number.

Although skill items are at the initial level of the assessment framework, they are not necessarily “easy” in all cases. Levels of reasoning and levels of difficulty are at times related (a Level 1: Skill item is more likely to be easier than a Level 3: Reasoning and justification item), but it is possible for a Level 1 item to be quite difficult for students (de Lange 1999, Webb 2002). Consider an item asking fifth-grade students to use the standard multiplication algorithm to compute 317 × 284. Although the steps in the procedures are repetitive and may be easy for some students, for many others, a problem involving that many digits may be challenging simply due to the sheer number of steps. Students may also have limited proficiency with the standard algorithm; therefore, using it may be almost too difficult. The same issue with difficulty can occur when recalling definitions. Consider our earlier example of asking students to define a prime number. If students know the definition, they simply need to recall the information from memory. But if a student has little experience with prime and composite numbers, the act of recalling a definition is not something they can do, and therefore, the item is quite challenging.

**Level 2: Problem solving and concepts**

The second level of the assessment framework comprises two distinct types of assessment items: problem-solving tasks and conceptual items. The key distinction is that Level 2: Problem-solving tasks require students to make sense of a realistic setting and determine what mathematics may apply to the situation. Level 2: Problem-solving tasks are often word problems for which students must determine the operation needed to solve the problem (Hiebert 1986).

Level 2: Concept items require the same depth of reasoning from students but do not necessarily include realistic situations (e.g., word problems). Concept tasks tend to focus on the use of mathematical models (e.g., number lines, arrays) or the application of mathematical properties (e.g., determining equivalent expressions using the order of operations).

In each case, Level 2 items are distinguishable from Level 1 items in that Level 2 tasks can have a variety of solution paths that the student must determine independently, and students are also expected to make decisions

**Level 3: Reasoning and justification**

These examples highlight the benefit of including multiple levels of assessment items in the classroom.

**FIGURE 1**

The IDMT assessment framework has three levels and four categories; Level 2 includes two different types of assessment.

- **Level 1: Skill**
  - Recall information, provide a definition, or perform a simple procedure

- **Level 2: Problem solving and concepts**
  - Problem solving—Solve a situation/context where you do not immediately know which solution path to take (or which skill or concept to apply), often because multiple ways exist to get to the answer
  - Concepts—Demonstrate, often visually, conceptual understanding of a topic

- **Level 3: Reasoning and justification**
  - Explain why an approach or solution is correct or incorrect, or why one method is more efficient for a given problem or set of tasks
based on their understanding of the content when solving Level 2 tasks.

**Level 3: Reasoning and justification**

Level 3 items require students to justify their own reasoning or critique the reasoning of others through careful analysis and use of mathematical properties. Level 3 items may also require the use of models and diagrams to support the response (de Lange 1999).

Some common types of Level 3 tasks are to ask students to explain why an approach or solution is correct or incorrect or why one method is more efficient for a given problem. Students’ responses can often make use of models and diagrams similar to those examined in Level 2: Concept items. The key distinction would be that in Level 3 tasks, students are challenged to think beyond their own personal interpretations of the content. They are asked to either explain their ideas in a way that would make sense to someone else or to reason about another person’s mathematical ideas. The requirements to demonstrate reasoning and then justify that reasoning make Level 3 items unique. Students are not necessarily afforded the opportunity to rely on concepts they know individually. The mathematical generalizations necessary in Level 3 items link these types of tasks to the essence of what mathematical proof is and also link the early development of mathematical properties and axioms when they emerge in later mathematical disciplines.

**Assessment writing and review**

To structure teachers’ assessment writing or review of assessment materials, institute staff offer teachers a matrix (see table 1). The example is from second grade and shows how teachers carefully considered each item on an end-of-unit test they intended to use to collect data on students’ learning of place value. Notice that each item was examined for the level of reasoning and the level of challenge (easy, moderate, or difficult) that was evident. In the example shown in table 1, only four items are on the quiz. If more items were needed, they would be added to the appropriate column.

**What does this type of assessment look like in the classroom?**

Following are two examples from classroom settings that further exemplify how the assessment framework informs teachers’ decisions about task selection and interactions with students.

**An example from first grade**

Kelli Rich teaches first grade in a linguistically diverse, high-needs school. Her students come to class with varied ability in mathematics and English language proficiency. She also teaches many students who may have attended only parts of kindergarten the year before.

During a unit on base-ten place value, Rich frequently made use of the assessment framework. Here is a brief vignette characterizing her interactions with students.

**Rich:** Nari, will you show me where the units of ten and units of one are in your model for twenty-three?

**Nari:** Here are the two units of ten [pointing to two drawn models of base-ten pieces], and here are the three units of one [pointing to three unit cubes].

**Rich:** I agree. I am wondering if you can find a way to draw a model for twenty-three that uses only one unit of ten or that doesn’t even use any tens. [This is an example of a Level 2: Concept task.]

**Nari:** Hmmm. I am not sure if that works.

**Rich:** Let’s use the base-ten pieces to investigate.
A matrix helps teachers structure assessment writing or review of materials. The four-item example below is from a second-grade end-of-unit quiz about place value. Teachers examined each item for level of reasoning and level of challenge.

### Sample assessment review matrix, grade 2

<table>
<thead>
<tr>
<th>Standards and key concepts</th>
<th>Level 1: Skill difficulty (E, M, D)</th>
<th>Level 2: Problem-solving difficulty (E, M, D)</th>
<th>Level 2: Concepts difficulty (E, M, D)</th>
<th>Level 3: Reasoning and justification difficulty (E, M, D)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Understand place value—specifically, the naming of place value locations and digits.</td>
<td>1. What digit is in the tens' place in the number 243? Easy</td>
<td>2. Pencils can be sold in boxes of 100, packs of 10, and as single pencils. Find three different combinations of boxes, packs, and singles for 318 pencils. Moderate</td>
<td>3. Using base-ten pieces, compose the number 345 in three different ways. Draw models of the base-ten pieces and label each unit of place value in your model to show your three different combinations. Moderate</td>
<td></td>
</tr>
<tr>
<td>Understand the relationship between different units of place value. For example, it requires 10 units of ten to compose 1 unit of one hundred. Accurately model place using concrete and visual models.</td>
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<tr>
<td>Understand place value as a prerequisite skill to learn to round numbers. Use models to communicate place value understanding. Skip count by units of place value.</td>
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<tr>
<td>4. Monica says 247 is closer to 200. Lupe says 247 is closer to 300. Who is correct and why? What is incorrect about the other student’s thinking? Use a number line to help support your answers. Difficult</td>
<td></td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>
After a brief exploration with the pieces, Nari determines that she could compose twenty-three using one unit of ten and thirteen units of one. Rich then challenges the students sitting within earshot of Nari to see whether they could find another way. Roger, another first grader, finds that he could compose twenty-three using all units of one.

The interaction among Rich, Nari, and the other students helps to illuminate the importance of teachers’ awareness of different levels of assessment. Rich could have simply noted that Nari was able to correctly model one way to compose twenty-three. But by pressing to a deeper level of reasoning, Rich extended Nari’s thinking and enhanced the learning of other students in the classroom. He also placed an emphasis on modeling the mathematics—Level 2: Concept items often require evidence to support mathematical ideas.

**An example from fourth grade**

Sarah Appleton is an instructional coach at an elementary school serving an ethnically diverse population predominantly from low-income families. She supports teachers by means of professional development, guided collaboration, and in-class assistance. Her school is currently working to adjust to the all-too-common challenge of increasing class size. Many of Appleton’s partner teachers have classrooms with little or no room for any additional desks and are struggling to avoid exceeding their maximum classroom capacity. As a result, Appleton spends much of her time assisting teachers in making decisions about instruction and assessment that also aid teachers in being efficient with their use of class time to assess student learning.

During a recent classroom visit, Appleton used the assessment framework to guide a class of fourth graders to a deeper level of analysis regarding equivalent fractions. Following is a brief vignette of this lesson.

**Appleton:** [to the classroom teacher] I am noticing that some students are struggling with the idea that two-fifths and four-tenths are the “same.” I have an idea we could try.

**Classroom teacher:** Yes! I will get their attention, and you can give them directions. I will jump in after I see what you’re doing. [*The classroom teacher directs the class to listen to Appleton.*]

**Appleton:** Class, I have heard some great discussions about the relationship between two-fifths and four-tenths. I am curious if you could help me explain this concept to some students I work with in another classroom. Maria, a student from another class, said that two-fifths and four-tenths are exactly the same, but Ivan told her that the two fractions are not exactly the same. What do you think each student meant, and how would you use a visual model to support your explanation? [This is an example of a Level 3: Reasoning and Justification task.]

After some brief discussion time between pairs of students, the classroom teacher and Appleton agree about several students who would be able to share some key ideas as well as visual models with the class. Maria, a student from another class, said that two-fifths and four-tenths are exactly the same, but Ivan told her that the two fractions are not exactly the same. What do you think each student meant, and how would you use a visual model to support your explanation? [This is an example of a Level 3: Reasoning and Justification task.]

After some brief discussion time between pairs of students, the classroom teacher and Appleton agree about several students who would be able to share some key ideas as well as visual models with the class. Some of these students help the class understand how two-fifths and four-tenths are at the same location on the number line and therefore must be equivalent. Another pair of students uses an area model to draw two-fifths first and then superimpose four-tenths over the two-fifths to show that the fractions are equal (see fig. 2). Finally, one student explains that although the fractions are equivalent, they are measured in different units (fifths and tenths), so something is slightly different.
The class eventually agrees that Maria’s idea is somewhat accurate but needs clarification about the subtle difference in the unit fractions.

This episode highlights the manner in which the assessment framework can be used to extend students’ knowledge in a setting in which the mathematical concepts being investigated are complex and nuanced. In this case, Appleton used her knowledge of the framework to help fourth-grade students understand that two-fifths and four-tenths are equivalent in the sense that they are the same value but also that they are slightly different in that they are representing the same value using different unit fractions. By modifying the task to press to this deeper level, both Appleton and the classroom teacher could more efficiently determine which students were able to reason mathematically about equivalent fractions in a more way and which students required more support.

**Immediate, specific intervention**

Assessment is one critical aspect of teaching within the larger set of mathematics teaching practices outlined in *Principles to Actions* (NCTM 2014). By using the IDMT assessment framework, teachers can thoroughly determine not only what students know but also how deeply students can reason about or understand the content. Students’ misconceptions may also become more evident, therefore giving teachers an opportunity to provide immediate interventions specific to the students’ demonstrated needs. This assessment framework becomes a valuable tool for all classroom teachers.

**REFERENCES**


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