Highlights from the National Mathematics Advisory Panel

The President established the National Mathematics Advisory Panel to examine the best available scientific research and make recommendations for improvements in the mathematics education of the nation's children. The Panel had a broad scope and reached many individual findings and recommendations, all conveyed in the main report. The highlights below contain very abbreviated versions of the most important points. The sections that did not pertain directly to classroom teaching were omitted.

Curricular Content

1) **A focused, coherent progression of mathematics learning, with an emphasis on proficiency with key topics, should become the norm in elementary and middle school mathematics curricula**. Any approach that continually revisits topics year after year without closure is to be avoided**.**

2) **The goal of an effective math program is to prepare the students to succeed in algebra**. To clarify instructional needs in Grades PreK–8 and to sharpen future discussion about the role of school algebra in the overall mathematics curriculum, the Panel developed a clear concept of school algebra via its list of Major Topics of School Algebra.

4) **A major goal for K–8 mathematics education should be proficiency with fractions (including decimals, percent, and negative fractions)**, for such proficiency is foundational for algebra and, at the present time, seems to be severely underdeveloped. Proficiency with whole numbers is a necessary precursor for the study of fractions, as are aspects of measurement and geometry. **These three areas—whole numbers, fractions, and particular aspects of geometry and measurement—are the Critical Foundations of Algebra.**

6) All school districts should ensure that all prepared students have access to an authentic algebra course—and should **prepare more students than at present to enroll in such a course by Grade 8.**

Learning Processes

10) **To prepare students for Algebra, the curriculum must simultaneously develop conceptual understanding, computational fluency, and problem- solving skills**. Debates regarding the relative importance of these aspects of mathematical knowledge are misguided. These capabilities are mutually supportive, each facilitating learning of the others. Teachers should emphasize these interrelations; taken together, conceptual understanding of mathematical operations, fluent execution of procedures, and fast access to number combinations jointly support effective and efficient problem solving.

11) **Computational proficiency with whole number operations is dependent on sufficient and appropriate practice to develop automatic recall of addition and related subtraction facts, and of multiplication and related division facts. It also requires fluency with the standard algorithms** for addition, subtraction, multiplication, and division. Additionally it requires a solid understanding of core concepts, such as the commutative, distributive, and associative properties. Although the learning of concepts and algorithms reinforce one another,

each is also dependent on different types of experiences, including practice.

12) **Difficulty with fractions (including decimals and percent) is pervasive and is a major obstacle to further progress in mathematics, including algebra**. A nationally representative sample of teachers of Algebra I who were surveyed for the Panel rated students as having very poor preparation in “rational numbers and operations involving fractions and decimals.” As with learning whole numbers, a conceptual understanding of fractions and decimals and the operational procedures for using them are mutually reinforcing. One key mechanism linking conceptual and procedural knowledge is the **ability to represent fractions on a number line.** The curriculum should afford sufficient time on task to ensure acquisition of conceptual and procedural knowledge of fractions and of proportional reasoning. Instruction focusing on conceptual knowledge of fractions is likely to have the broadest and largest impact on problem-solving performance when it is directed toward the accurate solution of specific problems.

13) Mathematics performance and learning of groups that have traditionally been underrepresented in mathematics fields can be improved by **interventions that address social, affective, and motivational factors**. Recent research documents that social and intellectual support from peers and teachers is associated with higher mathematics performance for all students, and that such support is especially important for many African-American and Hispanic students.

14) Children’s goals and beliefs about learning are related to their mathematics performance. Experimental studies have demonstrated that **changing children’s beliefs from a focus on ability to a focus on effort increases their engagement in mathematics learning, which in turn improves mathematics outcomes: When children believe that their efforts to learn make them “smarter,” they show greater persistence in mathematics learning**. Teachers and other educational leaders should consistently help students and parents to understand that an **increased emphasis on the importance of effort is related to improved mathematics performance**. This is a critical point because much of the public’s self-evident resignation about mathematics education (together with the common tendencies to dismiss weak achievement and to give up early) seems rooted in the erroneous idea that success is largely a matter of inherent talent or ability, not effort.

15) Teachers and developers of instructional materials sometimes assume that students need to be a certain age to learn certain mathematical ideas. However, a major research finding is that what is developmentally appropriate is largely contingent on prior opportunities to learn. **Claims based on theories that children of particular ages cannot learn certain content because they are “too young,” “not in the appropriate stage,” or “not ready” have consistently been shown to be wrong.** Nor are claims justified that children cannot learn particular ideas because their brains are insufficiently developed, even if they possess the prerequisite knowledge for learning the ideas.

Teachers and Teacher Education

17) **Research on the relationship between teachers’ mathematical knowledge and students’ achievement confirms the importance of teachers’ content knowledge**. It is self-evident that teachers cannot teach what they do not know. Direct assessments of

teachers' actual mathematical knowledge provide the strongest indication of a relation between teachers' content knowledge and their students' achievement.

Instructional Practices

23) **All-encompassing recommendations that instruction should be entirely "student centered" or "teacher directed" are not supported by research**. If such recommendations exist, they should be rescinded. If they are being considered, they should be avoided. High-quality research does not support the exclusive use of either approach.

24) Research has been conducted on a variety of cooperative learning approaches. **One such approach, Team Assisted Individualization (TAI), has been shown to improve students' computation skills**. This highly structured pedagogical strategy involves heterogeneous groups of students helping each other, individualized problems based on student performance on a diagnostic test, specific teacher guidance, and rewards based on both group and individual performance. Effects of TAI on conceptual understanding and problem solving were not significant.

25) **Teachers' regular use of formative assessment improves their students' learning, especially if teachers have additional guidance on using the assessment to design and to individualize instruction.** Although research to date has only involved one type of formative assessment (that based on items sampled from the major curriculum objectives for the year, based on state standards), the results are sufficiently promising that the Panel recommends regular use of formative assessment for students in the elementary grades.

26) The use of "real-world" contexts to introduce mathematical ideas has been advocated, with the term "real world" being used in varied ways**. A synthesis of findings from a small number of high-quality studies indicates that if mathematical ideas are taught using "real-world" contexts, then students' performance on assessments involving similar "real-world" problems is improved.** However, performance on assessments more focused on other aspects of mathematics learning, such as computation, simple word problems, and equation solving, is not improved.

27) **Explicit instruction with students who have mathematical difficulties has shown consistently positive effects on performance with word problems and computation**. Results are consistent for students with learning disabilities, as well as other students who perform in the lowest third of a typical class. By the term explicit instruction, the Panel means that teachers provide clear models for solving a problem type using an array of examples, that students receive extensive practice in use of newly learned strategies and skills, that students are provided with opportunities to think aloud (i.e., talk through the decisions they make and the steps they take), and that students are provided with extensive feedback. This finding does not mean that all of a student's mathematics instruction should be delivered in an explicit fashion. However, the Panel recommends that struggling students receive some explicit mathematics instruction regularly. Some of this time should be dedicated to ensuring that these students possess the foundational skills and conceptual knowledge necessary for understanding the mathematics they are learning at their grade level.

28) **Research on instructional software has generally shown positive effects on**

students' achievement in mathematics as compared with instruction that does not incorporate such technologies. These studies show that technology-based drill and practice and tutorials can improve student performance in specific areas of mathematics. Other studies show that teaching computer programming to students can support the development of particular mathematical concepts, applications, and problem solving. However, the nature and strength of the results vary widely across these studies. In particular, one recent large, multisite national study found no significant effects of instructional tutorial (or tutorial and practice) software when implemented under typical conditions of use. Taken together, the available research is insufficient for identifying the factors that influence the effectiveness of instructional software under conventional circumstances.

29) A review of 11 studies that met the Panel's rigorous criteria (only one study less than 20 years old) found limited or no impact of calculators on calculation skills, problem solving, or conceptual development over periods of up to one year. This finding is limited to the effect of calculators as used in the 11 studies. However, the Panel's survey of the nation's algebra teachers indicated that the use of calculators in prior grades was one of their concerns. **The Panel cautions that to the degree that calculators impede the development of automaticity, fluency in computation will be adversely affected**.

30) Mathematically gifted students with sufficient motivation appear to be able to learn mathematics much faster than students proceeding through the curriculum at a normal pace, with no harm to their learning, and should be allowed to do so